BRIEF EDITION

FOR BUSINESS, ECONOMICS, AND THE SOCIAL AND LIFE SCIENCES

CALCULUS

Eleventh Edition

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Calculus

For Business, Economics, and the Social and Life Sciences

BRIEF

Eleventh Edition

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For Business, Economics, and the Social and Life Sciences

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In memory of our parents
Doris and Banesh Hoffmann
and
Mildred and Gordon Bradley

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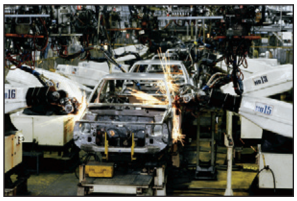

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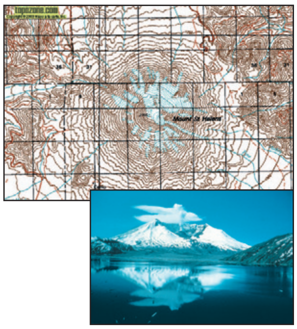


APPENDIX A Algebra Review

PREFACE

Overview of the Brief Eleventh Edition

Calculus for Business, Economics, and the Social and Life Sciences, Brief Edition, provides a sound, intuitive understanding of the basic concepts students need as they pursue careers in business, economics, and the life and social sciences. Students achieve success using this text as a result of the authors' applied and real-world orientation to concepts, problem-solving approach, straightforward and concise writing style, and comprehensive exercise sets. More than 100,000 students worldwide have studied from this text!

Improvements to This Edition

Revised Content

Every section in the text underwent careful analysis and extensive review to ensure the most beneficial and clear presentation. Additional steps and definition boxes were added when necessary for greater clarity and precision, graphs and figures were revised as necessary, and discussions and introductions were added or rewritten as needed to improve presentation.

Enhanced Topic Coverage

Material on the extreme value property for functions of two variables and finding extreme values on closed, bounded regions has been added to Section 7.3. This completes the analogy with the single-variable case and better prepares students for future study of statistics and finite mathematics.

Improved Exercise Sets

Almost 250 new routine and application exercises have been added to the already extensive problem sets. A wealth of new applied problems has been added to help demonstrate the practicality of the material, and existing applications have been updated. Moreover, exercise sets have been rearranged so that applications are grouped in categories (business/economics, life and social sciences, and miscellaneous).

New Pedagogical Design Elements

Titles have been added to each example in the text, and learning objectives have been specified at the beginning of each section. Example titles allow both students and instructors to quickly find items of interest to them. These pedagogical improvements make the topics clear and comprehensible for all students, help to organize ideas, and aid both students and professors with review and evaluation.

Online Matrix Supplement

The authors have fully revised the matrix supplement. Problems and examples have been revised and updated to include more contemporary applications. The revised supplement in PDF format is posted online for instructors to download at www.mhhe.com/hoffmann.

Chapter-by-Chapter Changes

- Titles have been added to all worked examples throughout the book.
- A list of learning objectives has been added at the beginning of every section.
- End-of-section exercises have been grouped according to subject.

Chapter 1

- New applied exercises have been added to Sections 1.1 through 1.5.
- Material in Section 1.2 on the rectangular coordinate system, the distance formula, intercepts, and quadratic functions has been added and rewritten. New and revised examples support these changes.
- In Section 1.4, the coverage of modeling has been revised and includes both new and revised examples.
- New notes, modified language, and new and revised examples in Section 1.5 help to clarify the topics of limits and infinity.
- A new example on break-even analysis has been added to Section 1.6.
- New Just-In-Time Reviews have been added to Sections 1.2 and 1.5.

Chapter 2

- The boxes for the constant multiple rule and sum rule have been rewritten to include the prime notation versions of the rules.
- Many introductions have been rewritten with an eye toward achieving better focus on describing a concept with greater clarity.
- Ten new exercises have been added to Section 2.3 (Exercises 36 through 39) and 2.4 (Exercises 61 through 64, 89, and 90).
- A new example using the chain rule twice has been added to Section 2.4.
- Section 2.5 includes a new introduction to marginal cost with a new example illustrating marginal cost and revenue. New exercises on marginal cost and revenue have also been added.
- A new introduction to implicit differentiation has been added, and there is a new Just-In-Time Review on related rates.

Chapter 3

- A new introductory example for increasing and decreasing functions has been added.
- There is a new discussion of worker efficiency and point of diminishing returns.
- The discussion and definition of inflection points and the box summarizing curve sketching with the second derivative have been modified.
- New exercises have been added in Sections 3.2 and 3.4.
- The material on price elasticity of demand has been completely rewritten.
- The chapter summary has been modified.

Chapter 4

- Boxes on the present and future values of an investment have been updated.
- New exercises on investment have been added in Section 4.1 and on elasticity of demand in Section 4.3.
- A new example with a new demand function has been added to Section 4.3.

Chapter 5

- In Section 5.1, there is a new introduction to differential equations and a new example on continuous compounding.
- There are 20 new exercises in Sections 5.1 and 5.3 (a total of 35 new exercises have been added throughout Chapter 5).

- A new subsection on the price-adjustment model in economics has been moved from Section 6.2 to Section 5.2, and new examples on price adjustment and a separable differential equation using substitution have been added to Section 5.2.
- There is a new introduction to Section 5.5.
- The table of Gini indices for various countries has been updated.
- The subsections on Consumer Willingness to Spend and Consumers' Surplus have been completely rewritten.

Chapter 6

- Old examples have been deleted from Section 6.1 in favor of a new applied example using the integral table to solve a logistic equation.
- Twenty-seven new exercises have been added to Chapter 6.
- A new introduction to improper integrals, new discussion and summary boxes for improper integrals involving $-\infty$, and a new example are included in Section 6.3.

Chapter 7

- Twenty-six new exercises have been added to Chapter 7.
- Data in Section 7.4 have been updated.
- Section 7.3 has been substantially revised. There is a new introduction to practical optimization, and a subsection involving optimization on a closed, bounded region (the extreme value property) has been added. This material helps students see how one- and two-dimensional optimization problems are related.
- A new subsection on finding population from population density has been added to Section 7.6.

KEY FEATURES OF THIS TEXT

Learning Objectives

1. Examine slopes of tangent lines and rates of change.
2. Define the derivative, and study its basic properties.
3. Compute and interpret a variety of derivatives using the definition.
4. Study the relationship between differentiability and continuity.

Learning Objectives

Each section begins with a list of objectives for that section. In addition to preparing students for what they will learn, these help students organize information for study and review and make connections between topics.

Applications

Throughout the text great effort is made to ensure that topics are applied to practical problems soon after their introduction, providing methods for dealing with both routine computations and applied problems. These problem-solving methods and strategies are introduced in applied examples and practiced throughout in the exercise sets.

"The example titles are a really excellent idea. From the student's perspective, they stimulate interest and get the students to read examples that interest them. From the instructor's perspective, they allow the instructor to make a decision about what to include without spending a lot of preparation time reading each example".

—*Jay Zimmerman, Towson University*

EXPLORE!

Refer to Example 3.5.4. Store

$$P(x) = \frac{75}{x} + \frac{300}{15 - x}$$

into Y1, and graph using the modified decimal window [0, 14]1 by [0, 350]10. Now use **TRACE** to move the cursor from X = 1 to 14 and confirm the location of minimal pollution. To view the behavior of the derivative $P'(x)$, enter Y2 = nDeriv(Y1, X, X) and graph using the window [0, 14]1 by [−75, 300]10. What do you observe?

EXAMPLE 3.5.4 Finding a Location That Minimizes Pollution

Two industrial plants, A and B, are located 15 miles apart and emit 75 ppm (parts per million) and 300 ppm of particulate matter, respectively. Each plant is surrounded by a restricted area of radius 1 mile in which no housing is allowed, and the concentration of pollutant arriving at any other point Q from each plant decreases with the reciprocal of the distance between that plant and Q. Where should a house be located on a road joining the two plants to minimize the total pollution arriving from both plants?

Solution

Suppose a house H is located x miles from plant A and, hence, $15 - x$ miles from plant B, where x satisfies $1 \leq x \leq 14$ since there is a 1-mile restricted area around each plant (Figure 3.49). Since the concentration of particulate matter arriving at H from each plant decreases with the reciprocal of the distance from the plant to H, the concentration of pollutant from plant A is $\frac{75}{x}$ and from plant B is $\frac{300}{15 - x}$. Thus, the total concentration of particulate matter arriving at H is given by the function

$$P(x) = \underbrace{\frac{75}{x}}_{\text{pollution from } A} + \underbrace{\frac{300}{15 - x}}_{\text{pollution from } B}$$

A General Procedure for Sketching the Graph of $f(x)$

Step 1. Find the domain of $f(x)$ [that is, where $f(x)$ is defined].

Step 2. Find and plot all intercepts. The y intercept (where $x = 0$) is usually easy to find, but the x intercepts [where $f(x) = 0$] may require a calculator.

Step 3. Determine all vertical and horizontal asymptotes of the graph. Draw the asymptotes in a coordinate plane.

Step 4. Find $f'(x)$, and use it to determine the critical numbers of $f(x)$ and intervals of increase and decrease.

Step 5. Determine all relative extrema (both coordinates). Plot each relative maximum with a cap (⌒) and each relative minimum with a cup (⌣).

Step 6. Find $f''(x)$, and use it to determine intervals of concavity and points of inflection. Plot each inflection point with a "twist" to suggest the shape of the graph near the point.

Step 7. You now have a preliminary graph, with asymptotes in place, intercepts plotted, arrows indicating the direction of the graph, and caps, cups, and twists suggesting the shape at key points. Plot additional points if needed, and complete the sketch by joining the plotted points in the directions indicated. Be sure to remember that the graph cannot cross a vertical asymptote.

Procedural Examples and Boxes

Each new topic is approached with careful clarity by providing step-by-step problem-solving techniques through frequent procedural examples and summary boxes.

Relative and Percentage Rates of Change ■ The **relative rate of change** of a quantity $Q(x)$ with respect to x is given by the ratio

$$\text{Relative rate of change of } Q(x) = \frac{Q'(x)}{Q(x)}$$

The corresponding **percentage rate of change** of $Q(x)$ with respect to x is

$$\text{Percentage rate of change of } Q(x) = \frac{100\,Q'(x)}{Q(x)}$$

Definitions

Definitions and key concepts are set off in shaded boxes to provide easy referencing for the student.

Just-In-Time REVIEW

In Example 1.5.6, we perform the multiplication

$$(\sqrt{x} - 1)(\sqrt{x} + 1) = x - 1$$

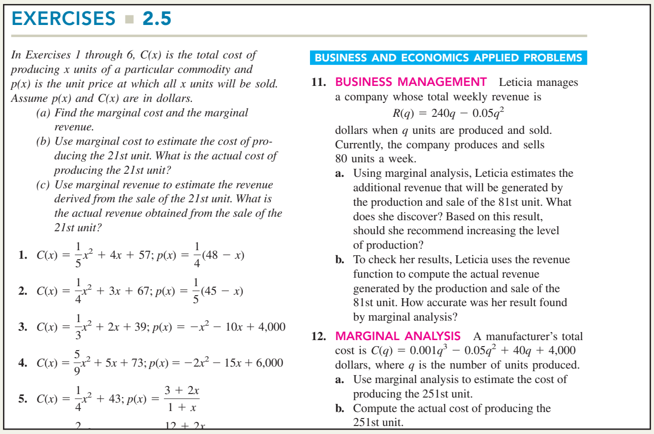
using the identity

$$(a - b)(a + b) = a^2 - b^2$$

with $\sqrt{x}$ corresponding to a and 1 corresponding to b.

Just-In-Time Reviews

These references, located in the margins, are used to quickly remind students of important concepts from college algebra or precalculus as they are being used in examples and review.



EXERCISES ■ 2.5

In Exercises 1 through 6, C(x) is the total cost of producing x units of a particular commodity and p(x) is the unit price at which all x units will be sold. Assume p(x) and C(x) are in dollars.

(a) Find the marginal cost and the marginal revenue.

(b) Use marginal cost to estimate the cost of producing the 21st unit. What is the actual cost of producing the 21st unit?

(c) Use marginal revenue to estimate the revenue derived from the sale of the 21st unit. What is the actual revenue obtained from the sale of the 21st unit?

1. $C(x) = \frac{1}{5}x^2 + 4x + 57; p(x) = \frac{1}{4}(48 - x)$
2. $C(x) = \frac{1}{4}x^2 + 3x + 67; p(x) = \frac{1}{5}(45 - x)$
3. $C(x) = \frac{1}{3}x^2 + 2x + 39; p(x) = -x^2 - 10x + 4{,}000$
4. $C(x) = \frac{5}{9}x^2 + 5x + 73; p(x) = -2x^2 - 15x + 6{,}000$
5. $C(x) = \frac{1}{4}x^2 + 43; p(x) = \frac{3 + 2x}{1 + x}$

BUSINESS AND ECONOMICS APPLIED PROBLEMS

11. BUSINESS MANAGEMENT Leticia manages a company whose total weekly revenue is
$$R(q) = 240q - 0.05q^2$$
dollars when q units are produced and sold. Currently, the company produces and sells 80 units a week.
   a. Using marginal analysis, Leticia estimates the additional revenue that will be generated by the production and sale of the 81st unit. What does she discover? Based on this result, should she recommend increasing the level of production?
   b. To check her results, Leticia uses the revenue function to compute the actual revenue generated by the production and sale of the 81st unit. How accurate was her result found by marginal analysis?

12. MARGINAL ANALYSIS A manufacturer's total cost is $C(q) = 0.001q^3 - 0.05q^2 + 40q + 4{,}000$ dollars, where q is the number of units produced.
   a. Use marginal analysis to estimate the cost of producing the 251st unit.
   b. Compute the actual cost of producing the 251st unit.

Exercise Sets

Almost 250 new problems have been added to increase the effectiveness of the highly praised exercise sets. Routine problems have been added where needed to ensure students have enough practice to master basic skills, and a variety of applied problems have been added to help demonstrate the practicality of the material.

Writing Exercises

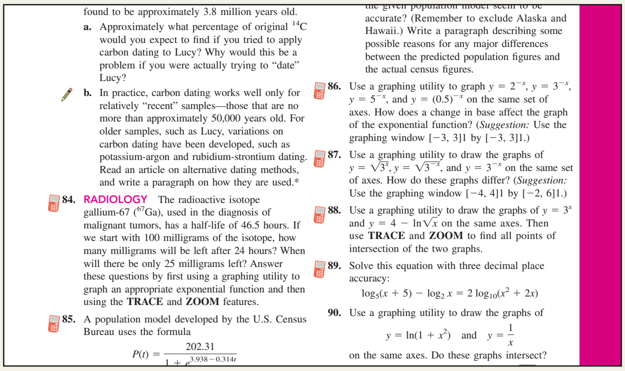
These problems, designated by writing icons, challenge a student's critical thinking skills and invite students to research topics on their own.

Calculator Exercises

Calculator icons designate problems within each section that can only be completed with a graphing calculator.

"[Hoffmann-Bradley] has excellent application problems in the social science, life science, economics, and finance fields."

—Rebecca Leefers, Michigan State University–East Lansing



found to be approximately 3.8 million years old.
   a. Approximately what percentage of original ^{14}C would you expect to find if you tried to apply carbon dating to Lucy? Why would this be a problem if you were actually trying to "date" Lucy?
   b. In practice, carbon dating works well only for relatively "recent" samples—those that are no more than approximately 50,000 years old. For older samples, such as Lucy, variations on carbon dating have been developed, such as potassium-argon and rubidium-strontium dating. Read an article on alternative dating methods, and write a paragraph on how they are used.*

84. RADIOLOGY The radioactive isotope gallium-67 (^{67}Ga), used in the diagnosis of malignant tumors, has a half-life of 46.5 hours. If we start with 100 milligrams of the isotope, how many milligrams will be left after 24 hours? When will there be only 25 milligrams left? Answer these questions by first using a graphing utility to graph an appropriate exponential function and then using the **TRACE** and **ZOOM** features.

85. A population model developed by the U.S. Census Bureau uses the formula
$$P(t) = \frac{202.31}{1 + e^{3.938 - 0.314t}}$$

accurate? (Remember to exclude Alaska and Hawaii.) Write a paragraph describing some possible reasons for any major differences between the predicted population figures and the actual census figures.

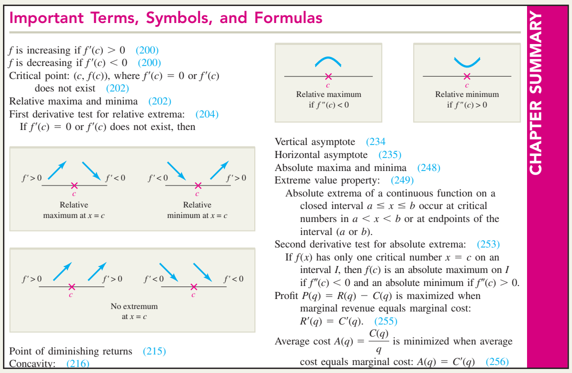
86. Use a graphing utility to graph $y = 2^{-x}$, $y = 3^{-x}$, $y = 5^{-x}$, and $y = (0.5)^{-x}$ on the same set of axes. How does a change in base affect the graph of the exponential function? (*Suggestion:* Use the graphing window $[-3, 3]1$ by $[-3, 3]1$.)

87. Use a graphing utility to draw the graphs of $y = \sqrt{3}^x$, $y = \sqrt{3}^{-x}$, and $y = 3^{-x}$ on the same set of axes. How do these graphs differ? (*Suggestion:* Use the graphing window $[-4, 4]1$ by $[-2, 6]1$.)

88. Use a graphing utility to draw the graphs of $y = 3^x$ and $y = 4 - \ln\sqrt{x}$ on the same axes. Then use **TRACE** and **ZOOM** to find all points of intersection of the two graphs.

89. Solve this equation with three decimal place accuracy:
$$\log_5(x + 5) - \log_2 x = 2\log_{10}(x^2 + 2x)$$

90. Use a graphing utility to draw the graphs of
$$y = \ln(1 + x^2) \quad \text{and} \quad y = \frac{1}{x}$$
on the same axes. Do these graphs intersect?



Important Terms, Symbols, and Formulas

CHAPTER SUMMARY

f is increasing if $f'(x) > 0$ (200)
f is decreasing if $f'(x) < 0$ (200)
Critical point: $(c, f(c))$, where $f'(c) = 0$ or $f'(c)$ does not exist (202)
Relative maxima and minima (202)
First derivative test for relative extrema: (204)
If $f'(c) = 0$ or $f'(c)$ does not exist, then

Point of diminishing returns (215)
Concavity: (216)

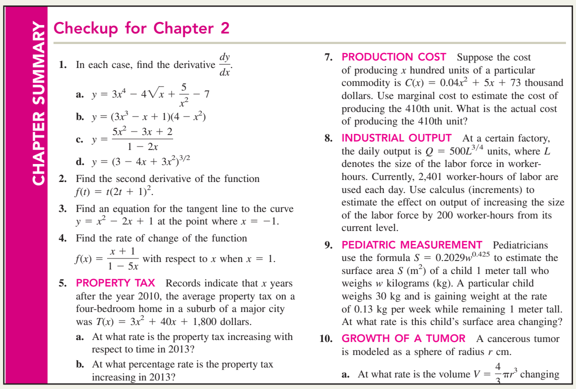
Vertical asymptote (234)
Horizontal asymptote (235)
Absolute maxima and minima (248)
Extreme value property: (249)
Absolute extrema of a continuous function on a closed interval $a \le x \le b$ occur at critical numbers in $a < x < b$ or at endpoints of the interval (a or b).
Second derivative test for absolute extrema: (253)
If $f(x)$ has only one critical number $x = c$ on an interval I, then $f(c)$ is an absolute maximum on I if $f''(c) < 0$ and an absolute minimum if $f''(c) > 0$.
Profit $P(q) = R(q) - C(q)$ is maximized when marginal revenue equals marginal cost: $R'(q) = C'(q)$. (255)
Average cost $A(q) = \frac{C(q)}{q}$ is minimized when average cost equals marginal cost: $A(q) = C'(q)$ (256)

Chapter Review

Chapter Review material aids the student in synthesizing the important concepts discussed within the chapter, including a master list of key technical terms and formulas introduced in the chapter.

Chapter Checkup

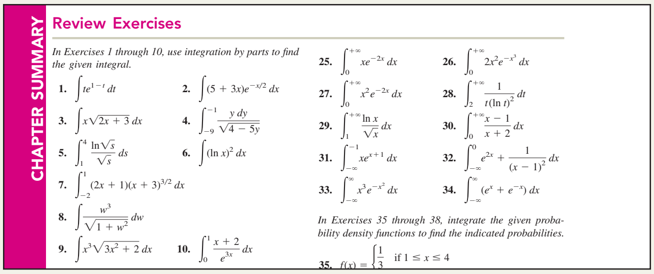
Chapter Checkups provide a quick quiz for students to test their understanding of the concepts introduced in the chapter.



CHAPTER SUMMARY

Checkup for Chapter 2

1. In each case, find the derivative $\frac{dy}{dx}$.
   a. $y = 3x^4 - 4\sqrt{x} + \frac{5}{x^2} - 7$
   b. $y = (3x^3 - x + 1)(4 - x^2)$
   c. $y = \frac{5x^2 - 3x + 2}{1 - 2x}$
   d. $y = (3 - 4x + 3x^2)^{3/2}$
2. Find the second derivative of the function $f(t) = t(2t + 1)^2$.
3. Find an equation for the tangent line to the curve $y = x^2 - 2x + 1$ at the point where $x = -1$.
4. Find the rate of change of the function $f(x) = \frac{x + 1}{1 - 5x}$ with respect to x when $x = 1$.
5. PROPERTY TAX Records indicate that x years after the year 2010, the average property tax on a four-bedroom home in a suburb of a major city was $T(x) = 3x^2 + 40x + 1{,}800$ dollars.
   a. At what rate is the property tax increasing with respect to time in 2013?
   b. At what percentage rate is the property tax increasing in 2013?
7. PRODUCTION COST Suppose the cost of producing x hundred units of a particular commodity is $C(x) = 0.04x^2 + 5x + 73$ thousand dollars. Use marginal cost to estimate the cost of producing the 410th unit. What is the actual cost of producing the 410th unit?
8. INDUSTRIAL OUTPUT At a certain factory, the daily output is $Q = 500L^{3/4}$ units, where L denotes the size of the labor force in worker-hours. Currently, 2,401 worker-hours of labor are used each day. Use calculus (increments) to estimate the effect on output of increasing the size of the labor force by 200 worker-hours from its current level.
9. PEDIATRIC MEASUREMENT Pediatricians use the formula $S = 0.2029w^{0.425}$ to estimate the surface area S (m^2) of a child 1 meter tall who weighs w kilograms (kg). A particular child weighs 30 kg and is gaining weight at the rate of 0.13 kg per week while remaining 1 meter tall. At what rate is this child's surface area changing?
10. GROWTH OF A TUMOR A cancerous tumor is modeled as a sphere of radius r cm.
   a. At what rate is the volume $V = \frac{4}{3}\pi r^3$ changing



CHAPTER SUMMARY

Review Exercises

In Exercises 1 through 10, use integration by parts to find the given integral.

1. $\int te^{1-t}\,dt$
2. $\int (5+3x)e^{-x/2}\,dx$
3. $\int x\sqrt{2x+3}\,dx$
4. $\int_{-9}^{-1} \frac{y\,dy}{\sqrt{4-5y}}$
5. $\int_1^4 \frac{\ln\sqrt{s}}{\sqrt{s}}\,ds$
6. $\int (\ln x)^2\,dx$
7. $\int_{-2}^{1} (2x+1)(x+3)^{3/2}\,dx$
8. $\int \frac{w^3}{\sqrt{1+w^2}}\,dw$
9. $\int x^3\sqrt{3x^2+2}\,dx$
10. $\int_0^1 \frac{x+2}{e^{3x}}\,dx$

25. $\int_0^{+\infty} xe^{-2x}\,dx$
26. $\int_0^{+\infty} 2x^2e^{-x^3}\,dx$
27. $\int_0^{+\infty} x^2e^{-2x}\,dx$
28. $\int_2^{+\infty} \frac{1}{t(\ln t)^2}\,dt$
29. $\int_1^{+\infty} \frac{\ln x}{\sqrt{x}}\,dx$
30. $\int_0^{+\infty} \frac{x-1}{x+2}\,dx$
31. $\int_{-\infty}^{-1} xe^{x+1}\,dx$
32. $\int_{-\infty}^{0} e^{2x} + \frac{1}{(x-1)^2}\,dx$
33. $\int_{-\infty}^{\infty} x^3e^{-x^2}\,dx$
34. $\int_{-\infty}^{\infty} (e^x + e^{-x})\,dx$

In Exercises 35 through 38, integrate the given probability density functions to find the indicated probabilities.

35. $f(x) = \begin{cases} \frac{1}{3} & \text{if } 1 \le x \le 4 \end{cases}$

Review Problems

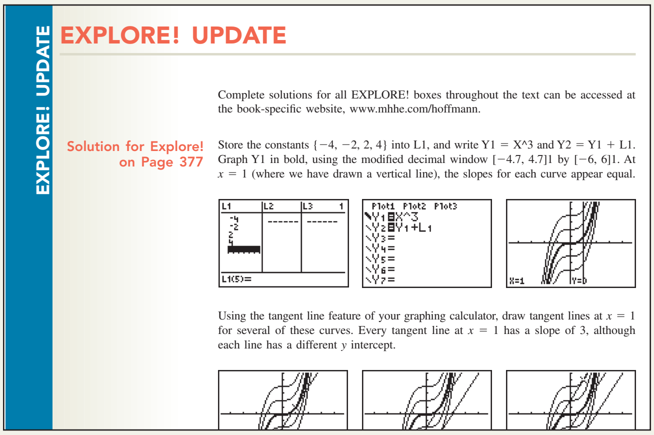
A wealth of additional routine and applied problems is provided within the end-of-chapter exercise sets, offering further opportunities for practice.

Explore! Technology

Utilizing the graphing calculator, Explore Boxes challenge a student's understanding of the topics presented with explorations tied to specific examples. Explore! Updates provide solutions and hints to selected boxes throughout the chapter.

"The book as a whole is one of the best calculus books I have used.... I really like how calculators are included on every section and that at the end of the chapter there is opportunity for students to explore the calculators even more."

—*Joseph Oakes, Indiana University Southeast*



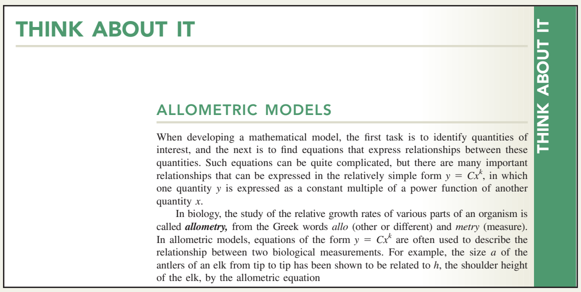
EXPLORE! UPDATE

Complete solutions for all EXPLORE! boxes throughout the text can be accessed at the book-specific website, www.mhhe.com/hoffmann.

Solution for Explore! on Page 377

Store the constants {−4, −2, 2, 4} into L1, and write Y1 = X^3 and Y2 = Y1 + L1. Graph Y1 in bold, using the modified decimal window [−4.7, 4.7]1 by [−6, 6]1. At $x = 1$ (where we have drawn a vertical line), the slopes for each curve appear equal.

Using the tangent line feature of your graphing calculator, draw tangent lines at $x = 1$ for several of these curves. Every tangent line at $x = 1$ has a slope of 3, although each line has a different y intercept.



THINK ABOUT IT

ALLOMETRIC MODELS

When developing a mathematical model, the first task is to identify quantities of interest, and the next is to find equations that express relationships between these quantities. Such equations can be quite complicated, but there are many important relationships that can be expressed in the relatively simple form $y = Cx^k$, in which one quantity y is expressed as a constant multiple of a power function of another quantity x.

In biology, the study of the relative growth rates of various parts of an organism is called ***allometry,*** from the Greek words *allo* (other or different) and *metry* (measure). In allometric models, equations of the form $y = Cx^k$ are often used to describe the relationship between two biological measurements. For example, the size a of the antlers of an elk from tip to tip has been shown to be related to h, the shoulder height of the elk, by the allometric equation

Think About It Essays

The modeling-based Think About It essays show students how material introduced in the chapter can be used to construct useful mathematical models while explaining the modeling process and providing an excellent starting point for projects or group discussions.

Also available . . .

Applied Calculus for Business, Economics, and the Social and Life Sciences, Expanded Eleventh Edition

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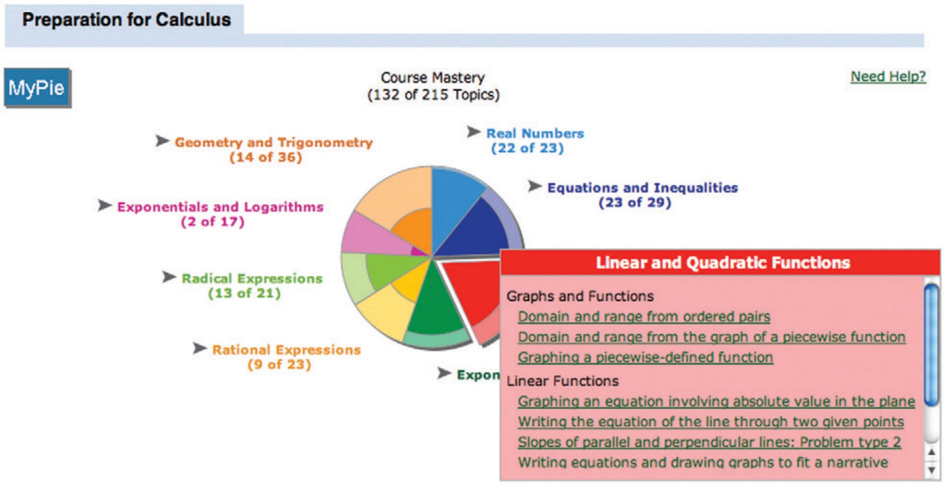

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Calculus

For Business, Economics, and the Social and Life Sciences

CHAPTER 1

Supply and demand determine the price of stock and other commodities.

Functions, Graphs, and Limits

SECTION 1.1 Functions

Learning Objectives

1. Identify the domain of a function, and evaluate a function from an equation.
2. Gain familiarity with piecewise-defined functions.
3. Introduce and illustrate functions used in economics.
4. Form and use composite functions in applied problems.

The word *function* is often used conversationally in connection with the act of playing a role, as seen in the following statements obtained in a Google search for the string "is a function of":

Just-In-Time REVIEW

Appendices A.1 and A.2 contain a brief review of algebraic properties needed in calculus.

"Intelligence is a function of experience."

"Human population is a function of food supply."

"Freedom is a function of economics."

What these statements have in common is that some quantity or characteristic (intelligence, population, freedom) depends on another (experience, food supply, economics). This is the essence of the mathematical concept of function.

Loosely speaking, a function consists of two sets and a rule that associates elements in one set with elements in the other. For instance, suppose you want to determine the effect of price on the number of iPods that can be sold at that price. To study this relationship, you need to know the acceptable prices, the set of possible sales levels, and a rule for associating each price with a particular sales level. Here is the definition we will use for function.

Function ■ A **function** is a rule that assigns to each object in a set A exactly one object in a set B. The set A is called the **domain** of the function, and the set of assigned objects in B is called the **range.**

For most functions we will consider, the domain and range will be collections of real numbers, and the function itself will be denoted by a letter like f. The value that the function f assigns to a number x in its domain is then denoted by $f(x)$. This is read as "f of x" (*never* as "f times x"). In many cases, we will use a formula like $f(x) = x^2 + 4$ to describe the value of a function.

You can also think of a function as a mapping from numbers in the domain set A to numbers in the range set B (Figure 1.1a) or as a machine that takes a given number from A and converts it into a specific number in B through a process prescribed

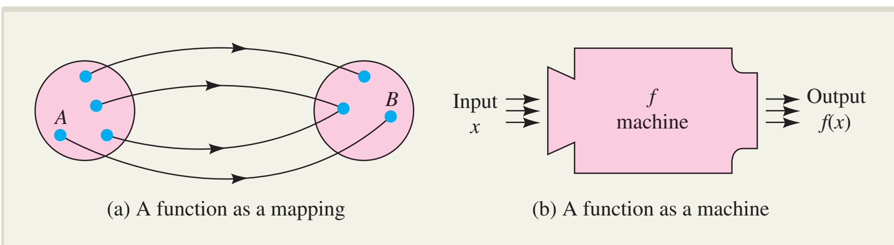


FIGURE 1.1 Interpretations of the function f.

by the functional rule (Figure 1.1b). Thus, the function $f(x) = x^2 + 4$ can be regarded as an f machine that accepts an input x, squares it, and then adds 4 to produce an output $y = x^2 + 4$.

Regardless of how you choose to think of a functional relationship, it is important to remember that *a function assigns one and only one number in the range (output) to each number in the domain (input)*. Example 1.1.1 illustrates the convenience of functional notation.

EXPLORE!

Store $f(x) = x^2 + 4$ into your graphing utility. Evaluate at $x = -3, -1, 0, 1$, and 3. Make a table of values. Repeat using $g(x) = x^2 - 1$. Explain how the values of $f(x)$ and $g(x)$ differ for each x value.

EXAMPLE 1.1.1 Evaluating a Function

Find and simplify $f(-3)$ if $f(x) = x^2 + 4$.

Solution

We interpret $f(-3)$ to mean "replace all x values in the formula for the function whose name is f with the number -3." Thus, we write

$$f(-3) = (-3)^2 + 4 = 13$$

Note the efficiency of this notation. In Example 1.1.1 the compact formula $f(x) = x^2 + 4$ completely defines the function, and you can indicate that 13 is the unique number the function assigns to -3 by simply writing $f(-3) = 13$.

It is often convenient to represent a functional relationship by an equation $y = f(x)$, and in this context, x and y are called **variables.** In particular, since the numerical value of y is determined by that of x, we refer to y as the **dependent variable** and to x as the **independent variable.** There is nothing sacred about the symbols x and y. For example, the function $y = x^2 + 4$ can just as easily be represented by $s = t^2 + 4$ or by $w = u^2 + 4$. These formulas are equivalent because in each the independent variable is squared and the result is increased by 4 to produce the value for the dependent variable.

Functional notation can also be used to describe tabular data. For instance, Table 1.1 lists the average tuition and fees for private 4-year colleges at 5-year intervals from 1973 to 2008.

TABLE 1.1 Average Tuition and Fees for 4-Year Private Colleges

Academic Year Ending in	Period p	Tuition and Fees T
1973	1	$1,898
1978	2	$2,700
1983	3	$4,639
1988	4	$7,048
1993	5	$10,448
1998	6	$13,785
2003	7	$18,273
2008	8	$25,177

We can describe these data as a function T whose rule is "assign to each value of p the average tuition and fees in dollars, $T(p)$, at the beginning of the pth 5-year period." Thus, $T(1) = \$1{,}898$, $T(2) = \$2{,}700, \ldots, T(8) = \$25{,}177$. Note that in this example we departed from the traditional function named f and independent variable named x. Instead, we chose T to represent the function name because it is suggestive of "tuition" just as using p for the independent variable is suggestive of "period."

In the absence of additional directions or restrictions, we will assume that the domain of a function f is the set of all numbers x for which $f(x)$ is defined. Thus, the domain of the function in Example 1.1.1 is the set of all real numbers since any number x can be squared and added to 4. On the other hand, the college tuition function T illustrated in Table 1.1 has the set of numbers $\{1, 2, \ldots, 8\}$ as its domain since $T(p)$ is given (defined) only for inputs $p = 1, 2, 3, \ldots, 8$. Here is the definition we will follow for the domain convention.

Domain Convention ■ Unless otherwise specified, we assume the domain of a function f to be all real numbers x for which $f(x)$ is defined as a real number. We refer to this as the **natural domain** of f.

Determining the natural domain of a function often amounts to excluding all inputs that result in dividing by 0 or in taking the square root of a negative number, as illustrated in Examples 1.1.2 and 1.1.3.

EXPLORE!

Store $f(x) = 1/(x - 3)$ in your graphing utility as Y1, and display its graph using a **ZOOM** Decimal Window. **TRACE** values of the function from X = 2.5 to 3.5. What do you notice at X = 3?

EXAMPLE 1.1.2 Finding the Domain of a Function

Find the domain of each of the functions.

a. $f(x) = \dfrac{1}{x - 3}$ **b.** $g(t) = \dfrac{\sqrt{3 - 2t}}{t^2 + 4}$

Solution

a. Because division by any number other than 0 is possible, the domain of f is the set of all numbers x such that $x - 3 \neq 0$; that is, $x \neq 3$.

b. The denominator $t^2 + 4$ of $g(t)$ is always positive, so we need not be concerned with dividing by 0. However, all numbers t such that $3 - 2t < 0$ must be excluded from the domain to prevent taking the square root of a negative number. Thus, the domain is the set of all numbers t such that $3 - 2t \geq 0$; that is, $t \leq \dfrac{3}{2}$.

***Just-In-Time* REVIEW**

Recall that $x^{a/b} = \sqrt[b]{x^a}$ whenever a and b are positive integers. Example 1.1.3 uses the case when $a = 1$ and $b = 2$; $x^{1/2}$ is another way of expressing $\sqrt{x}$.

EXAMPLE 1.1.3 Evaluating an Applied Function

A satellite TV company commissions a study that finds the number of customers who can be accommodated each hour by its customer service call center is given by the function $N(w) = 30(w - 1)^{1/2}$, where w is the number of workers at the center. Find $N(5)$, $N(17)$, $N(1)$, and $N(0)$, and interpret your results.

Solution

First, rewrite the function as $N(w) = 30\sqrt{w-1}$. (Fractional exponents are discussed in Appendix A.1 if you need a quick review.) Then

$$N(5) = 30\sqrt{5-1} = 30\sqrt{4} = 30(2) = 60$$
$$N(17) = 30\sqrt{17-1} = 30\sqrt{16} = 30(4) = 120$$
$$N(1) = 30\sqrt{1-1} = 30(0) = 0$$

but $N(0)$ is not defined since $30\sqrt{0-1} = 30\sqrt{-1}$ and negative numbers do not have real square roots.

This tells us that the call center can accommodate 60 callers per hour with 5 workers, 120 callers per hour with 17 workers, and no callers with only 1 worker. It also tells us that 0 workers is not an acceptable input for this function.

Functions are often defined using more than one formula, where each individual formula describes the function on a subset of the domain. A function defined in this way is sometimes called a **piecewise-defined function.** Such functions appear often in business, biology, and physics applications. In Example 1.1.4, we use a piecewise-defined function to describe sales.

EXPLORE!

Create a simple piecewise-defined function using the boolean algebra features of your graphing utility. Store $Y1 = 2(X < 1) + (-1)(X \geq 1)$ in the function editor. Examine the graph of this function, using the **ZOOM** decimal window. What values does Y1 assume at $X = -2, 0, 1,$ and 3?

EXAMPLE 1.1.4 Evaluating a Piecewise-Defined Function

Suppose we use a function to model the stock price over time of Deckers Outdoor Corporation, the company that produces the popular Ugg boots. While Uggs have been on the market since 1979, during 2003 Ugg sales, and consequently stock values, increased dramatically. It makes sense to use one formula to model stock prices before 2003 and another to model it afterward. Let $S(t)$ represent the stock price of Deckers Outdoor Corporation t years after January 1, 2000. Then

$$S(t) = \begin{cases} 8.1 - 1.7t & \text{if } t < 3 \\ 6t^2 - 36t + 57 & \text{if } t \geq 3 \end{cases}$$

Find and interpret $S(2)$, $S(3)$, and $S(7.5)$.

Solution

Because $t = 2$ satisfies $t < 3$, we use the first formula to calculate the value of the function. Then $S(2) = 8.1 - 1.7(2) = 4.7$. In terms of the model, this means that on January 1, 2002, the share price of Deckers Outdoor Corporation was predicted to be \$4.70.

Both $t = 3$ and $t = 7.5$ satisfy $t \geq 3$, so we use the second formula to evaluate $S(3)$ and $S(7.5)$. We find that

$$S(3) = 6(3)^2 - 36(3) + 57 = 3$$

and

$$S(7.5) = 6(7.5)^2 - 36(7.5) + 57 = 124.5$$

Therefore, share prices were predicted to be \$3 per share on January 1, 2003, and \$124.50 per share on July 1, 2007, the day 7.5 years after January 1, 2000.

Functions Used in Economics

We will study several functions associated with the marketing of a particular commodity.

The **demand function** $D(x)$ for the commodity is the price $p = D(x)$ that must be charged for each unit of the commodity if x units are to be sold (demanded).

The **supply function** $S(x)$ for the commodity is the unit price $p = S(x)$ at which producers are willing to supply x units to the market.

The **revenue** $R(x)$ obtained from selling x units of the commodity is given by the product

$$\begin{aligned} R(x) &= \text{(number of items sold)(price per item)} \\ &= xp(x) \end{aligned}$$

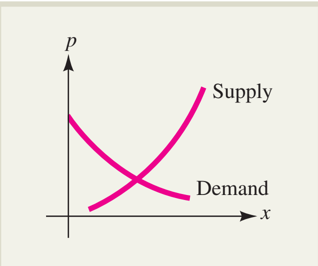
The **cost function** $C(x)$ is the cost of producing x units of the commodity.

The **profit function** $P(x)$ is the profit obtained from selling x units of the commodity and is given by the difference

$$\begin{aligned} P(x) &= \text{revenue} - \text{cost} \\ &= R(x) - C(x) = xp(x) - C(x) \end{aligned}$$

The **average cost function** is $AC(x) = \dfrac{C(x)}{x}$. Similarly, the average revenue function $AR(x)$ and average profit function $AP(x)$ are given by

$$AR(x) = \frac{R(x)}{x} \quad \text{and} \quad AP(x) = \frac{P(x)}{x}$$



Generally speaking, the higher the unit price, the fewer the number of units demanded, and vice versa. Similarly, an increase in unit price leads to an increase in the number of units supplied. Thus, demand functions are typically decreasing ("falling" from left to right), while supply functions are increasing ("rising"), as illustrated in the margin. Example 1.1.5 uses several of these special economic functions.

EXAMPLE 1.1.5 Studying a Production Process

Market research indicates that consumers will buy x thousand units of a particular kind of coffee maker when the unit price is

$$p(x) = -0.27x + 51$$

dollars. The cost of producing the x thousand units is

$$C(x) = 2.23x^2 + 3.5x + 85$$

thousand dollars.

a. What is the average cost of producing 4,000 coffee makers?

b. How much revenue $R(x)$ and profit $P(x)$ are obtained from producing x thousand units (coffee makers)?

c. For what values of x is production of the coffee makers profitable?

Solution

a. A production level of 4,000 coffee makers corresponds to $x = 4$ (since x is in thousands of units), and the corresponding average cost is

$$AC(4) = \frac{C(4)}{4} = \frac{2.23(4)^2 + 3.5(4) + 85}{4}$$
$$= \frac{134.68}{4} = 33.67 \text{ thousand dollars per thousand units}$$

So the average cost is \$33.67 per coffee maker produced.

b. The revenue is the price $p(x)$ times the number of units x:

$$R(x) = xp(x) = -0.27x^2 + 51x$$

thousand dollars. The profit is the revenue minus the cost:

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.27x^2 + 51x - (2.23x^2 + 3.5x + 85) \\ &= -2.5x^2 + 47.5x - 85 \end{aligned}$$

thousand dollars.

c. Production is profitable when the profit function has a positive output, that is, when $P(x) > 0$. First, we factor the profit function:

$$\begin{aligned} P(x) &= -2.5x^2 + 47.5x - 85 \\ &= -2.5(x^2 - 19x + 34) \\ &= -2.5(x - 2)(x - 17) \end{aligned}$$

Since -2.5 is negative, the profit $P(x) = -2.5(x - 2)(x - 17)$ is positive only when the product $(x - 2)(x - 17)$ is also negative. This happens when the separate factors $x - 2$ and $x - 17$ have opposite signs. Since there are no x values for which $x - 2 < 0$ and $x - 17 > 0$, we must have $x - 2 > 0$ and $x - 17 < 0$, that is, $2 < x < 17$. So production is profitable when the level of production is between 2,000 and 17,000 units.

Just-In-Time REVIEW

The product of two numbers is positive if they have the same sign and negative if they have different signs. That is, $ab > 0$ if $a > 0$ and $b > 0$ and also if $a < 0$ and $b < 0$. On the other hand, $ab < 0$ if $a < 0$ and $b > 0$ or if $a > 0$ and $b < 0$.

Example 1.1.6 provides another illustration of functional notation in a practical situation. Once again, the letters assigned for the function and the independent variable are suggestive of the real quantities they represent.

EXPLORE!

Refer to Example 1.1.6 and store the cost function $C(q)$ into Y1 as

$$X^3 - 30X^2 + 500X + 200$$

Construct a **TABLE** of values for $C(q)$ using your calculator, setting TblStart at X = 5 with an increment ΔTbl = 1 unit. On the table of values observe the cost of manufacturing the 10th unit.

EXAMPLE 1.1.6 Evaluating a Cost Function

Suppose the total cost in dollars of manufacturing m treadmills is given by the function $C(m) = m^3 - 30m^2 + 500m + 200$.

a. Find the cost of manufacturing 10 treadmills. What is the average cost of producing these treadmills?

b. Compute the cost of manufacturing the 10th treadmill.

Solution

a. The cost of manufacturing 10 treadmills is the value of the total cost function when $m = 10$; that is,

$$\text{Cost of 10 treadmills} = C(10) = (10)^3 - 30(10)^2 + 500(10) + 200 = 3{,}200$$

The average cost of producing the 10 treadmills is

$$AC(10) = \frac{C(10)}{10} = \frac{3{,}200}{10} = 320$$

So the total cost of producing 10 treadmills is \$3,200, and the average cost is \$320 per treadmill.

b. The cost of manufacturing the 10th treadmill is the difference between the cost of manufacturing 10 treadmills and the cost of manufacturing 9 treadmills:

$$\text{Cost of 10th treadmill} = C(10) - C(9) = 3{,}200 - 2{,}999 = \$201$$

Composition of Functions

There are many situations in which a quantity is given as a function of one variable that, in turn, can be written as a function of a second variable. By combining the functions in an appropriate way, you can express the original quantity as a function of the second variable. This process is called **composition of functions** or **functional composition.**

For instance, consider a factory that produces GPS units. The number of units produced depends on the amount of material available which, in turn, depends on the amount of capital spent on material. So overall, the production level depends on the amount of capital spent on material. In this sense, production is a composite function of capital expenditure. Here is a definition of functional composition.

Composition of Functions ■ Given functions $f(u)$ and $g(x)$, the composition $f(g(x))$ is the function of x formed by substituting $u = g(x)$ for u in the formula for $f(u)$.

Note that the composite function $f(g(x))$ makes sense only if the domain of f contains the range of g. In Figure 1.2, the definition of composite function is illustrated as an assembly line in which raw input x is first converted into a transitional product $g(x)$ that acts as input the f machine uses to produce $f(g(x))$.

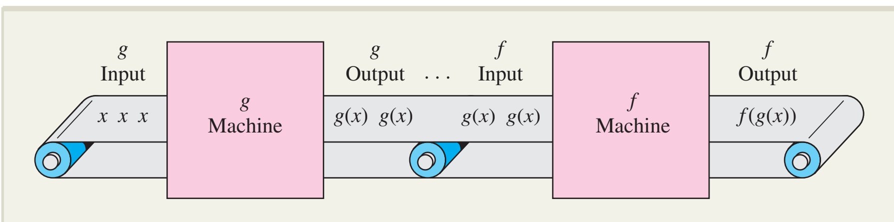


FIGURE 1.2 The composition $f(g(x))$ as an assembly line.

The construction of a composite function is illustrated in Example 1.1.7.

EXAMPLE 1.1.7 Finding a Composite Function

Find the composite function $f(g(x))$, where $f(u) = u^2 + 3u + 1$ and $g(x) = x + 1$.

Solution

Replace u by $x + 1$ in the formula for $f(u)$ to get

$$\begin{aligned} f(g(x)) &= (x + 1)^2 + 3(x + 1) + 1 \\ &= (x^2 + 2x + 1) + (3x + 3) + 1 \\ &= x^2 + 5x + 5 \end{aligned}$$

EXPLORE!

Store the functions $f(x) = x^2$ and $g(x) = x + 3$ into Y1 and Y2, respectively, of the function editor. Deselect (turn off) Y1 and Y2. Set Y3 = Y1(Y2) and Y4 = Y2(Y1). Show graphically (using **ZOOM** Standard) and analytically (by table values) that $f(g(x))$ represented by Y3 and $g(f(x))$ represented by Y4 are not the same functions. What are the explicit equations for both of these composites?

NOTE By reversing the roles of f and g in the definition of composite function, you can define the composition $g(f(x))$. In general, $f(g(x))$ and $g(f(x))$ will *not* be the same. For instance, with the functions in Example 1.1.7, you first write

$$g(w) = w + 1 \qquad \text{and} \qquad f(x) = x^2 + 3x + 1$$

and then replace w by $x^2 + 3x + 1$ to get

$$\begin{aligned} g(f(x)) &= (x^2 + 3x + 1) + 1 \\ &= x^2 + 3x + 2 \end{aligned}$$

which is quite different from $f(g(x)) = x^2 + 5x + 5$ found in Example 1.1.7. In fact, $f(g(x)) = g(f(x))$ only when

$$\begin{aligned} x^2 + 5x + 5 &= x^2 + 3x + 2 \\ 2x &= -3 \\ x &= -\frac{3}{2} \end{aligned}$$

■

Example 1.1.7 could have been worded more compactly as follows: Find the composite function $f(x + 1)$ where $f(x) = x^2 + 3x + 1$. The use of this compact notation is illustrated further in Example 1.1.8.

EXAMPLE 1.1.8 Expressing Cost as a Composite Function

Neal, the owner of a small furniture company, finds that if r recliners are produced per hour, the cost will be $C(r)$ dollars, where

$$C(r) = r^3 - 50r + \frac{1}{r + 1}$$

Suppose, in turn, the production level satisfies $r = 4 + 0.3w$, where w is the hourly wage of the workers.

a. Express the cost of production as a composite function of hourly wage.

b. How much should Neal expect to pay for production when workers earn \$20 per hour?